3.21–3.25 Review Key

Name_____ Date _____ Period _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question. SHOW WORK FOR ALL PROBLEMS!

Write the sum or difference in the standard form a + bi.

1) (9 − 5i) + (8 + 9i) 1) _____

$9+8=17$
$-5i+9i=4i$
$17+4i$

2) (4 + 5i) − (−2 + i) 2) _____

$4-(-2)=6$
$5i-i=4i$
$6+4i$

Write the product in standard form.

3) $(\sqrt{15}+9i)(\sqrt{15}-9i)$ 3) _____

$15-9\sqrt{15}i+9\sqrt{15}i-81i^2 = 15+81 = 96$

Write the expression in standard form.

4) $\frac{9+4i}{4-6i} \cdot \frac{4+6i}{4+6i} = \frac{36+54i+16i+24i^2}{4^2+6^2} = \frac{12+70i}{52}$ 4) _____

$= \frac{12}{52} + \frac{70}{52}i = \frac{3}{13} + \frac{35}{26}i$

Find the product of the complex number and its conjugate.

5) $(-2-5i)(-2+5i) = (-2)^2+(5)^2 = 4+25 = 29$ 5) _____

Write the expression in the form a + bi, where a and b are real numbers.

6) $\frac{\sqrt{-20}\cdot\sqrt{-4}}{\sqrt{5}} = \frac{2\sqrt{5}i \cdot 2i}{\sqrt{5}} = 4i^2 = -4$ 6) _____

Find the absolute value of the complex number. Round your answer to two decimal places, if necessary.

7) 2 − 4i $|2-4i| = \sqrt{(2)^2+(-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ 7) _____

Write the complex number in trigonometric form, using degree measure for the argument.

8) 6 + 8i $r = \sqrt{(6)^2+(8)^2} = \sqrt{36+64} = \sqrt{100} = 10$ 8) _____

$\theta = \tan^{-1}\left(\frac{8}{6}\right) \approx 53.1°$

$10(\cos 53.1° + i\sin 53.1°)$

Write the complex number in the form a + bi.

9) $\sqrt{6}(\cos 315° + i \sin 315°)$ 9) _____

$\sqrt{6}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \frac{2\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}i = \boxed{\sqrt{3} - \sqrt{3}i}$

Perform the indicated operation. Write the answer in the form a + bi.

10) $4(\cos 135° + i \sin 135°) \cdot 6(\cos 225° + i \sin 225°)$ 10) _____

$4 \cdot 6\left(\cos(135° + 225°) + i\sin(135° + 225°)\right)$

$24(\cos 360° + i\sin 360°) = 24(1 + 0i) = \boxed{24}$

11) $\dfrac{8(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2})}{3(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6})}$ 11) _____

$= \frac{8}{3}\left(\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\right)$

$= \frac{8}{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = \frac{8}{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= \boxed{\frac{4}{3} + \frac{4\sqrt{3}}{3}i}$

Use De Moivre's theorem to simplify the expression. Write the answer in a + bi form.

12) $(2(\cos 45° + i \sin 45°))^3$ 12) _____

$2^3\left(\cos(3 \cdot 45°) + i\sin(3 \cdot 45°)\right) = 8(\cos 135° + i\sin 135°)$

$= 8\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \boxed{-4\sqrt{2} + 4\sqrt{2}i}$

13) $(2 - 2i)^5$ 13) _____

$r = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = -45°$

$(2\sqrt{2})^5\left(\cos(5 \cdot (-45)) + i\sin(5 \cdot (-45))\right)$

$= 128\sqrt{2}\left(\cos(-225°) + i\sin(-225°)\right) = 128\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \boxed{-128 + 128i}$

Find the indicated roots. Write the answer in trigonometric form.

14) Cube roots of $125(\cos 312° + i \sin 312°)$ 14) _____

$n = 3$

$r = 125^{1/3} = 5$

$\theta_1 = \frac{312}{3} = 104°$

$\frac{360}{3} = 120°$

$z_1 = 5(\cos 104° + i\sin 104°)$

$z_2 = 5(\cos 224° + i\sin 224°)$

$z_3 = 5(\cos 344° + i\sin 344°)$

Find all specified roots. write answer in a+bi form.

15) Cube roots of 8

$n=3$

$r = 8^{1/3} = 2$

$\theta_1 = \frac{0°}{3} = 0°$

$\frac{360}{3} = 120°$

$z_1 = 2(\cos 0° + i \sin 0°) = 2(1 + 0i) = \boxed{2}$ 15) _____

$z_2 = 2(\cos 120° + i \sin 120°) = 2(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) = \boxed{-1 + \sqrt{3}i}$

$z_3 = 2(\cos 240° + i \sin 240°) = 2(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = \boxed{-1 - \sqrt{3}i}$

Solve the equation. Write the answer in a + bi form.

16) $x^2 + 8 = 0$ 16) _____

$n=2$

$r = 8^{1/2} = 2\sqrt{2}$

$\theta = 180°$

$\frac{\theta}{2} = 90°$

$\frac{360}{2} = 180°$

$x^2 = -8$ square roots of -8

$x_1 = 2\sqrt{2}(\cos 90° + i \sin 90°) = 2\sqrt{2}(0 + i) = \boxed{2\sqrt{2}i}$

$x_2 = 2\sqrt{2}(\cos 270° + i \sin 270°) = 2\sqrt{2}(0 - i) = \boxed{-2\sqrt{2}i}$

Convert the rectangular coordinates to polar coordinates, using radian measure for the angle.

17) $(8, -8)$ 17) _____

4th Quad.

$r = \sqrt{(8)^2 + (-8)^2} = \sqrt{64 + 64} = \sqrt{128} = 8\sqrt{2}$

$\theta = \tan^{-1}(\frac{-8}{8}) = -\pi/4 + 2\pi$

$\boxed{(8\sqrt{2}, \frac{7\pi}{4})}$

Convert to rectangular coordinates.

18) $\left(-4, -\frac{\pi}{3}\right)$ 18) _____

$x = -4\cos(-\frac{\pi}{3}) = -4(\frac{1}{2}) = -2$

$y = -4\sin(-\pi/3) = -4(-\frac{\sqrt{3}}{2}) = 2\sqrt{3}$

$\boxed{(-2, 2\sqrt{3})}$

For the point given in rectangular coordinates, find equivalent polar coordinates (r, θ) for r > 0 and 0° ≤ θ < 360°.

19) $(-4, 4\sqrt{3})$ 19) _____

2nd Quad.

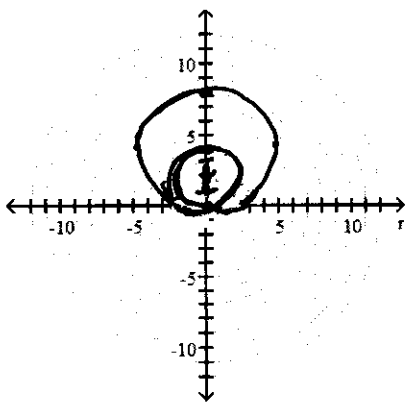
$r = \sqrt{(-4)^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = \sqrt{64} = 8$

$\theta = \tan^{-1}(\frac{4\sqrt{3}}{-4}) = -60° + 180° = 120°$

$\boxed{(8, 120°)}$

Graph the polar equation.

20) $r = 2(1 + 3 \sin \theta)$ 20) _____



For the given polar equation, write an equivalent rectangular equation.

21) $r = 10 \sin \theta$ 21) _____

$r^2 = 10r\sin\theta$

$x^2 + y^2 = 10y$

Using the pair of parametric equations, find the values of A and B in the table.

22) $x = 3t + 7$, $y = t + 8$, for $0 \le t \le 7$ 22) _____

t	x	y
0		
A	19	
2		B

$19 = 3A + 7$

$12 = 3A$

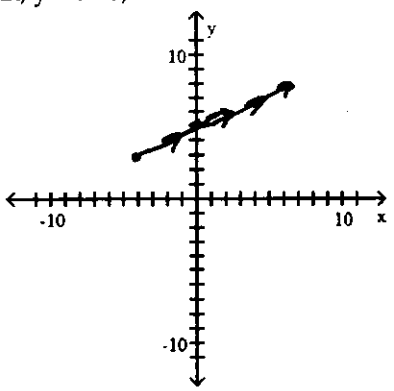
$4 = A$

$B = 2 + 8$

$B = 10$

Graph the pair of parametric equations in the rectangular coordinate system.

23) $x = 2t$, $y = t + 5$; $-2 \le t \le 3$ 23) _____



t	$x = 2t$	$y = t+5$
-2	-4	3
-1	-2	4
0	0	5
1	2	6
2	4	7
3	6	8

Eliminate the parameter of the pair of parametric equations.

24) $x = t - 3$, $y = t^2 + 5$ 24) _____

$(x+3) = t$ $\quad y = (x+3)^2 + 5$

$y = x^2 + 6x + 9 + 5$

$y = x^2 + 6x + 14$

Solve the problem.

25) A projectile is fired with an initial velocity of 300 feet per second at an angle of 45° with the horizontal. To the nearest foot, find the maximum altitude of the projectile. The parametric equations for the path of the projectile are 25) _____

$x = (300 \cos 45°)t$, and

$y = (300 \sin 45°)t - 16t^2$.

Graph $y = 300\sin(45°)x - 16x^2$

In rectangular mode and find maximum.

max altitude: 703 ft.